UNIVERSITY OF SALERNO, ITALY

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The Role of Symmetry in Finding the Equivalent Resistance of Regular Networks

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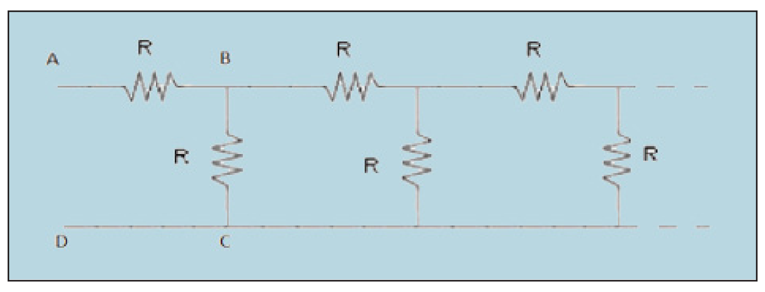
Abstract

The topic concerning the role of symmetry in calculating the equivalent resistance of regular networks has been addressed to the students of Lyceum "Imbriani" in Avellino ad of Lyceum "De Caprariis" in Atripalda, in two successive years. These activities were part of the PLS project carried out at University of Salerno. The calculation and meansurement methods adopted show that subject can be easily introduced as a lecture and as a laboratory session in High School; after having introduced the traditional concept of equivalent resistance of series and parallel connections of resistors.

Introduction

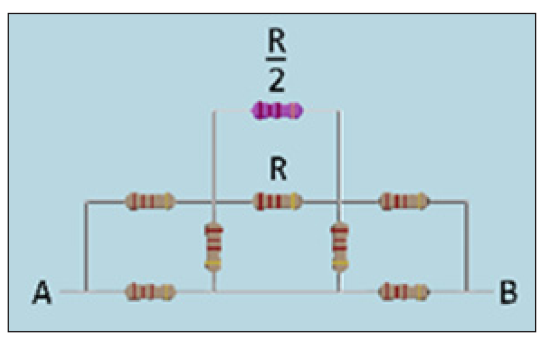
The concept of equivalent resistance in series and parallel connections of resistors is rarely extended to more complex networks in High School physics lectures. However, it is possible to show that those concepts can be extended to find the equivalent resistance of networks possessing specific geometrical properties. In this project we have shown that, by making use of particular symmetries of regular networks, the equivalent resistance, as measured by connecting two leads to two nodes of the network itself, can be calculated with no much more effort than in usual cases. The role of symmetry in infinite networks shaped as regular platonic or archimedean solids has been highlighted.

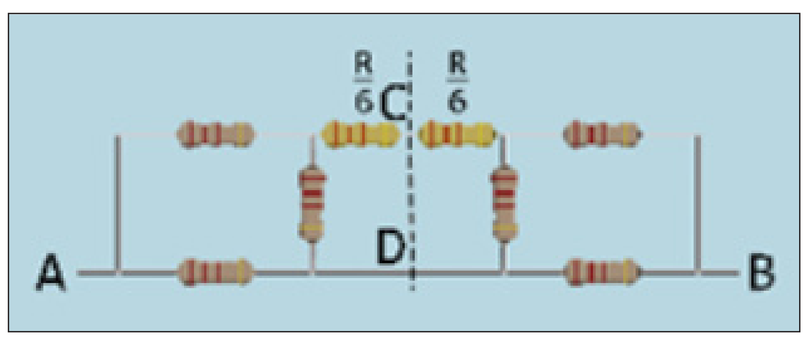
Infinite Networks

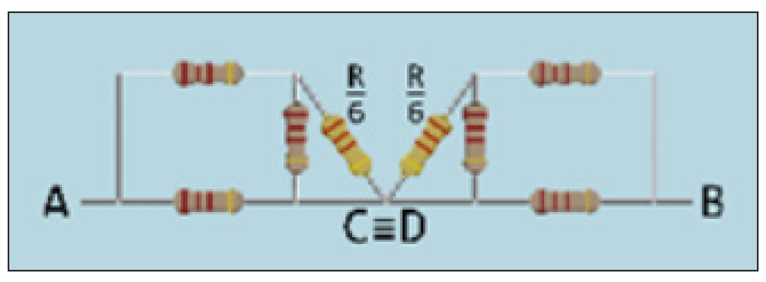


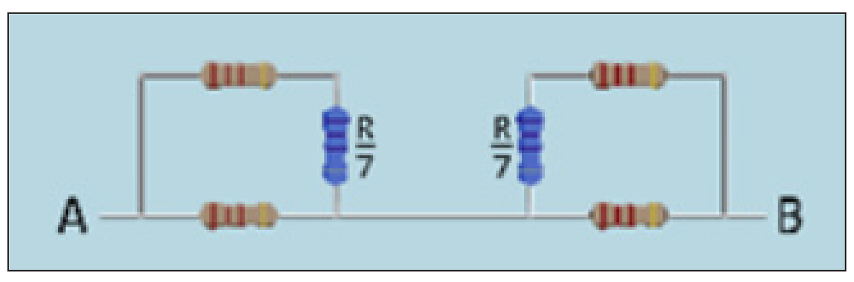
With this work we addressed the problem of searching for the equivalent resistance between two nodes of bi-dimensional networks mainly using the symmetric properties of the circuit. The network can be symmetrical with respect to a point "central symmetry", with respect to a line "axial symmetry" and with respect to a plane "mirror symmetry".

Axial Symmetry

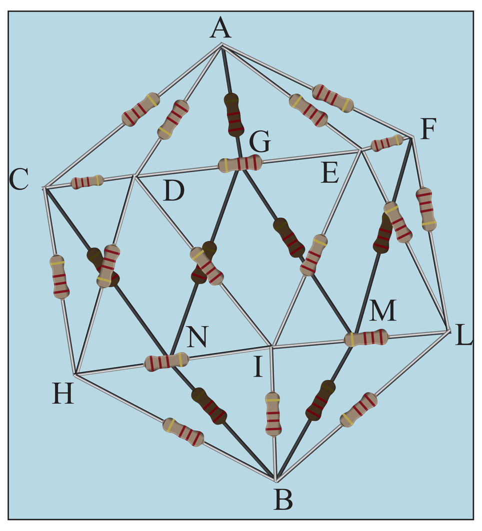






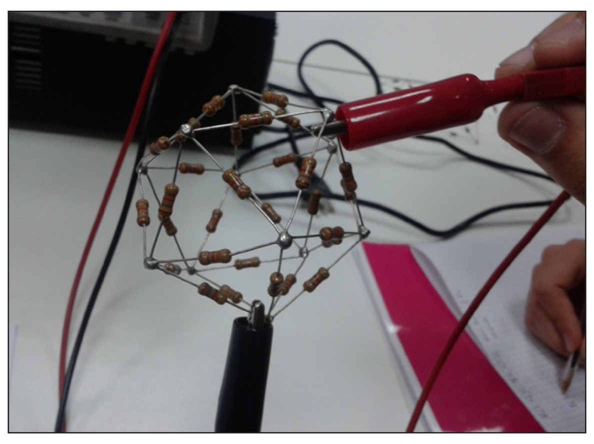


Platonic Solids



If we put a electromotive force between the points A and B of the solid, the points C, D, E, F, G (V_1) and the points H, I, L, M, N (V_2) are at the same voltage. Both the first and second group of vertices can be considered electrically as a macro node, thus being connected to the same nodes, the resistances of these branches CH – HD – DI – IE – FL – FM – MG – NG – NC are, as a result, in parallel. Branches AC – AD – AE – AF – AG and branches BH – BL – BN are in parallel. The equivalent resistance is R/2 and is the sum of the 3 in series resistances with value R/5, R/10 and R/5.

The Laboratory



As a first step, students constructed three-dimensional models of platonic solids using thin cardboard. Subsequently, they numbered the edges, so that they could implement Laplace's method and calculated the matrix ratio with the software Microsoft Excel. They then built paper models also for some Archimedean solids: in this way, they could experiment the validity of the calculation method, already used for platonic solids. As a second step, students fabricated Platonic and Archimedean solids in the lab, using identical resistances of 2200 Ω, 1800 Ω, 1200 Ω, 22000 Ω and 10 Ω. Lastly, they measured the equivalent resistances (R_{eq}) between some of the vertices of the three-dimensional networks and verified the theoretical model.

Solid	Resistances		Methods		
	R(Ω)	vertices	Laplace	Dutch	Experimental
Tetrahedron	(2200± 5%) Ω	R_{eq} = (1, 2)	(1100 ± 5%) Ω	(1100 ± 5%) Ω	(1097,5 ± 0,1) Ω
Cube	(1800± 5%) Ω	R_{eq} = (1, 2)	(1050 ± 5%) Ω	(1050 ± 5%) Ω	(1050,0 ± 0,1) Ω
		R_{eq} = (4, 8)	(1350 ± 5%) Ω		(1349,2 ± 0,1) Ω
		R_{eq} = (1, 5)	(1500 ± 5%) Ω		(1500,0 ± 0,1) Ω
Octahedron	(1200± 5%) Ω	R_{eq} = (1, 2)	(600 ± 5%) Ω	(600 ± 5%) Ω	(598,6 ± 0,1) Ω
		R_{eq} = (1, 4)	(500 ± 5%) Ω		(497,0 ± 0,1) Ω
Icosahedron	(1800± 5%) Ω	R_{eq} = (1, 2)	(660 ± 5%) Ω	(660 ± 5%) Ω	(662,0 ± 0,1) Ω
		R_{eq} = (1, 8)	(900 ± 5%) Ω		(904,6 ± 0,1) Ω
		R_{eq} = (1, 5)	(840 ± 5%) Ω		(842,0 ± 0,1) Ω
Dodecahedron	(2200± 5%) Ω	R_{eq} = (1, 2)	(1393 ± 5%) Ω	(1393 ± 5%) Ω	(1386,5 ± 0,1) Ω
		R_{eq} = (1, 14)	(2567 ± 5%) Ω		(2568,8 ± 0,1) Ω
		R_{eq} = (1, 3)	(1980 ± 5%) Ω		(1977,0 ± 0,1) Ω
		R_{eq} = (1, 18)	(2493 ± 5%) Ω		(2499,4 ± 0,1) Ω
		R_{eq} = (1, 17)	(2347 ± 5%) Ω		(2348,9 ± 0,1) Ω
Cuboctahedron	(22000± 5%) Ω	R_{eq} = (1, 4)	(10083 ± 5%) Ω		(10123,0 ± 0,1) Ω
		R_{eq} = (1, 12)	(14667 ± 5%) Ω		(14706,0 ± 0,1) Ω
Truncatred Octahedron	(10,0± 5%) Ω	R_{eq} = (1, 2)	(6,2 ± 5%) Ω		(6,2 ± 0,1) Ω
		R_{eq} = (7, 8)	(6,8 ± 5%) Ω		(6,8 ± 0,1) Ω
Truncatred tetrahedron	(10,0± 5%) Ω	R_{eq} = (1, 2)	(7,0 ± 5%) Ω		(6,9 ± 0,1) Ω
		R_{eq} = (1, 7)	(5,7 ± 5%) Ω		(5,7 ± 0,1) Ω
		R_{eq} = (1, 12)	(11,0 ± 5%) Ω		(10,9± 0,1) Ω

Conclusions

The theoretical values calculated in the table are affected by the error of 5% due to the value of the resistances used. The experimental results are , therefore, within the limits of error, comparable with the theoretical model of the proposed method.