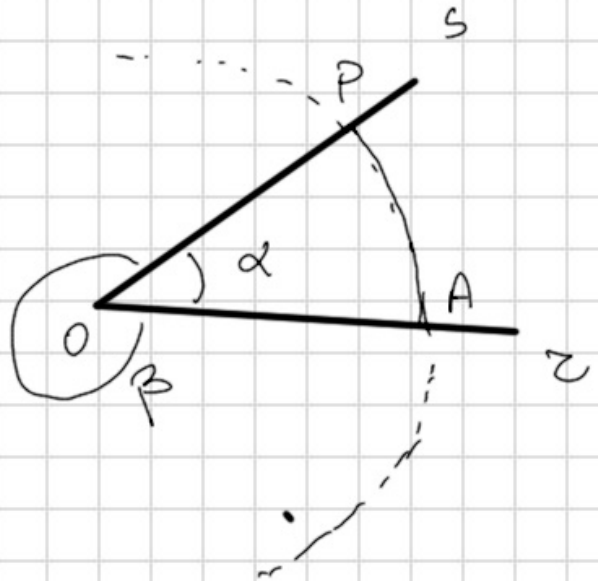
Lezione del 7-12-2023 TRIGONOMETRIA

ANGOLO



$\overline{OA} = R$

$\overset{\frown}{AP}$

$$\alpha = \frac{\overset{\frown}{AP}}{R}$$

CIRCONFERENZA $2\pi R$ $\quad \alpha = 2\pi$ GIRO

SEMICIRCONF. πR $\quad \alpha = \pi$ PIATTO

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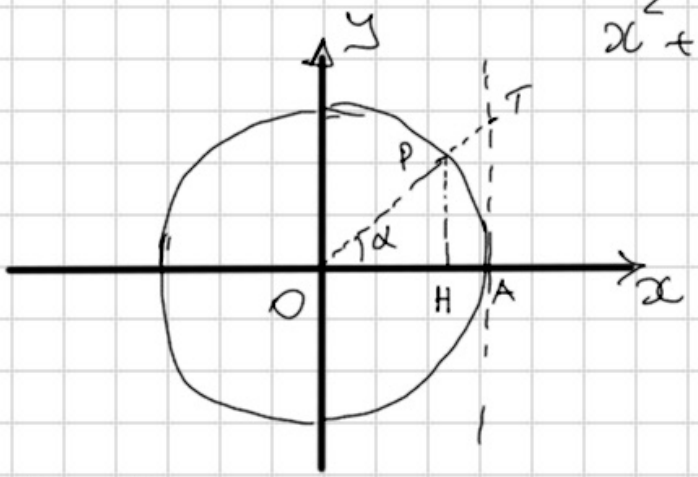
GRADI	0	30	45	60	90	180	270	360
RADIANTI	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	π	$\frac{3}{2}\pi$	2π

$\alpha_{rad} : \pi = \alpha^{\circ} : 180$



$x^2 + y^2 = 1$

$\sin\alpha = \overline{PH}$

$\cos\alpha = \overline{OH}$

$\frac{\overline{AT}}{\overline{OA}} = tg\,\alpha = \frac{\overline{PH}}{\overline{OH}}$

$\frac{\sin\alpha}{\cos\alpha} = tg\,\alpha$

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gradi	$0°$	$30°$	$45°$	$60°$	$90°$	$180°$	$270°$	$360°$
radianti	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	π	$\frac{3}{2}\pi$	2π
$\sin\alpha$	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1	0	-1	0
$\cos\alpha$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0	-1	0	1
$\tan\alpha$	0	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$	∞	0	∞	0
$\cot\alpha$	∞	$\sqrt{3}$	1	$\frac{\sqrt{3}}{3}$	0	∞	0	∞

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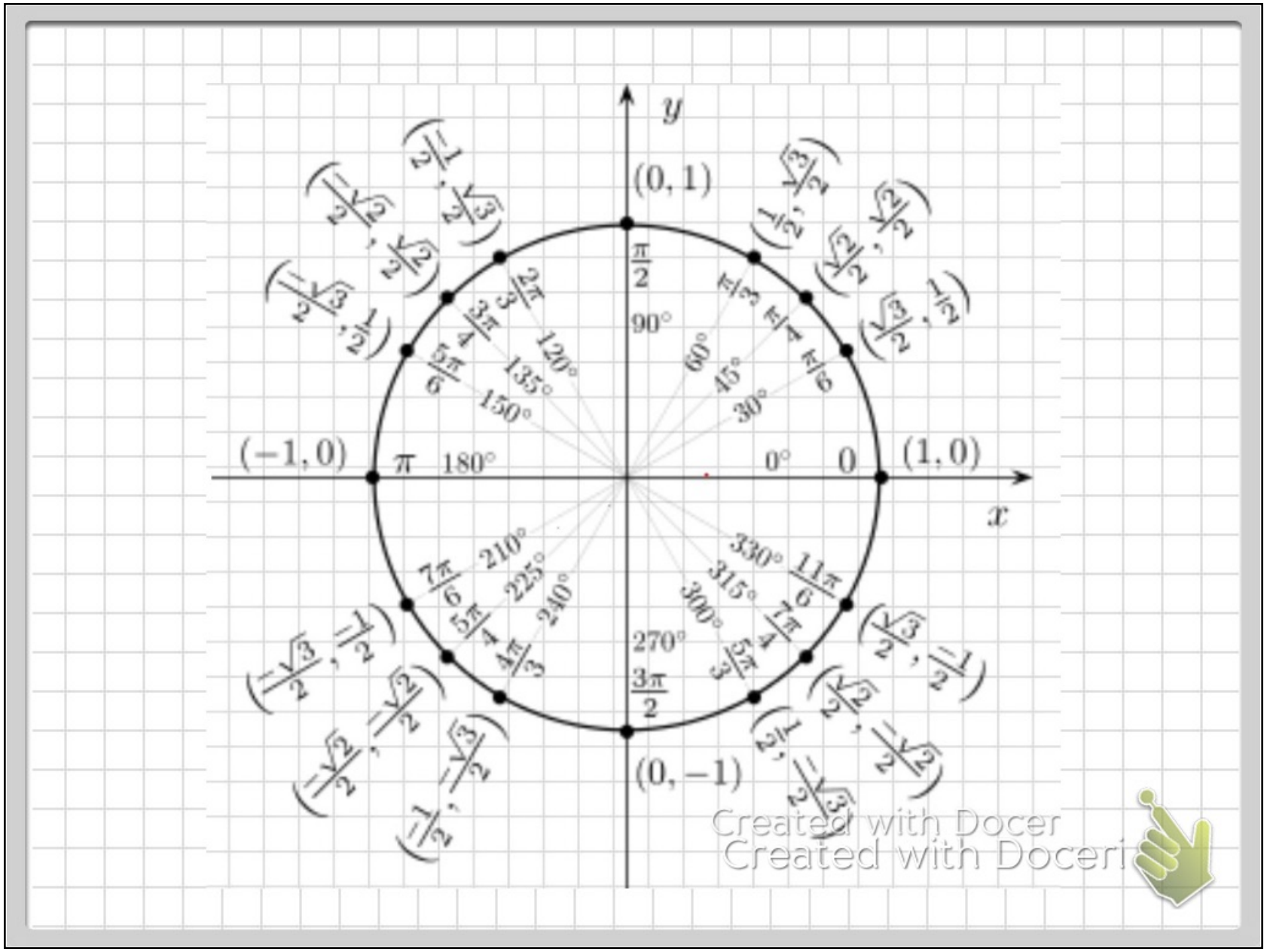
y
x
(0, 1)
(1, 0)
(−1, 0)
(0, −1)
90°
60°
45°
30°
0°
0
120°
135°
150°
180°
π
210°
225°
240°
270°
300°
315°
330°
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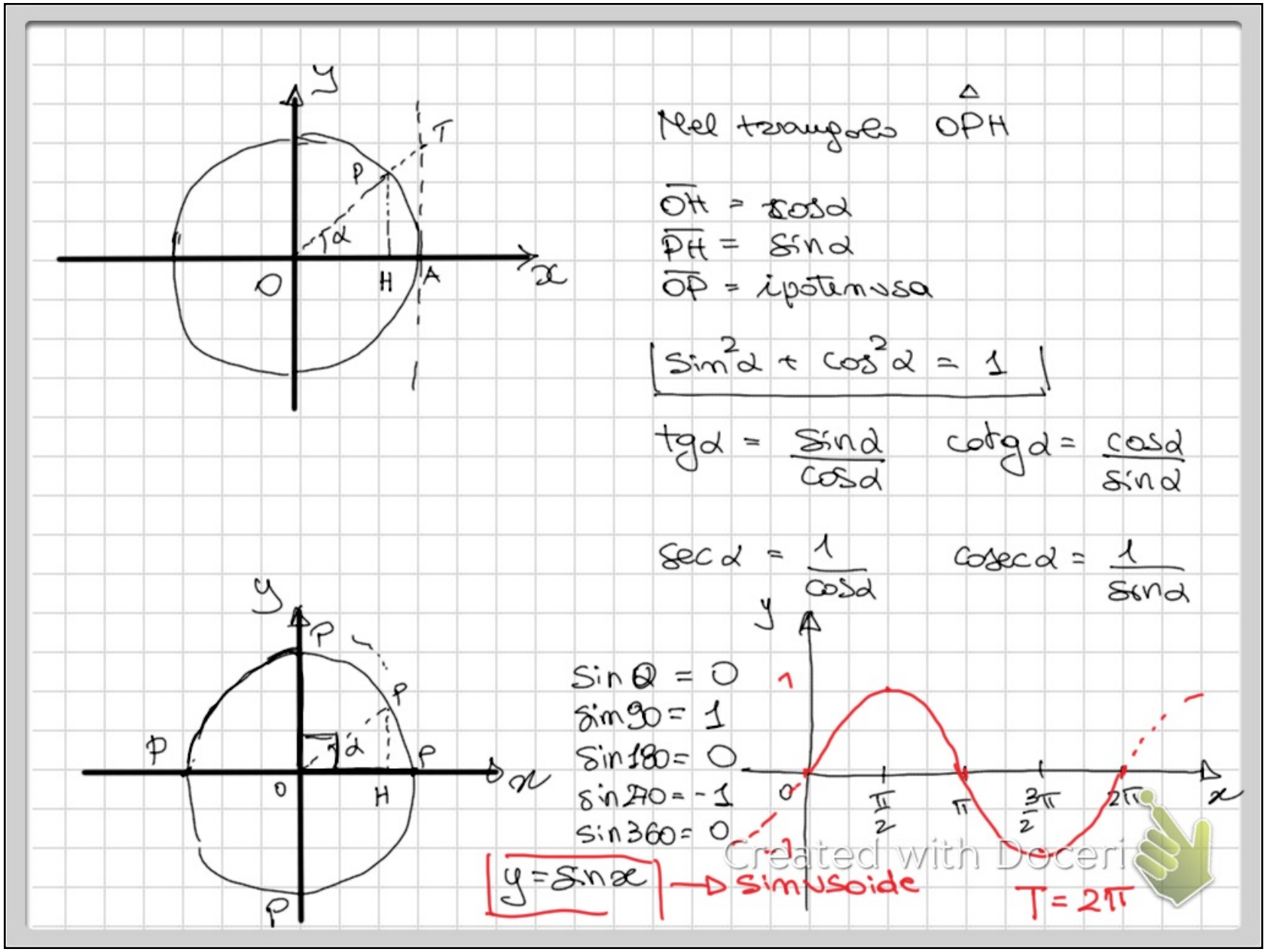
y
T
P
α
O
H
A
x
Nel triangolo OPH
$\overline{OH} = \cos\alpha$
$\overline{PH} = \sin\alpha$
$\overline{OP}$ = ipotenusa
$\sin^2\alpha + \cos^2\alpha = 1$
$\operatorname{tg}\alpha = \frac{\sin\alpha}{\cos\alpha}$
$\operatorname{cotg}\alpha = \frac{\cos\alpha}{\sin\alpha}$
$\sec\alpha = \frac{1}{\cos\alpha}$
$\operatorname{cosec}\alpha = \frac{1}{\sin\alpha}$
y
P
P
P
P
α
O
H
x
$\sin 0 = 0$
$\sin 90 = 1$
$\sin 180 = 0$
$\sin 270 = -1$
$\sin 360 = 0$
y
1
0
$\frac{\pi}{2}$
π
$\frac{3\pi}{2}$
2π
x
$y = \sin x$ → sinusoide
$T = 2\pi$
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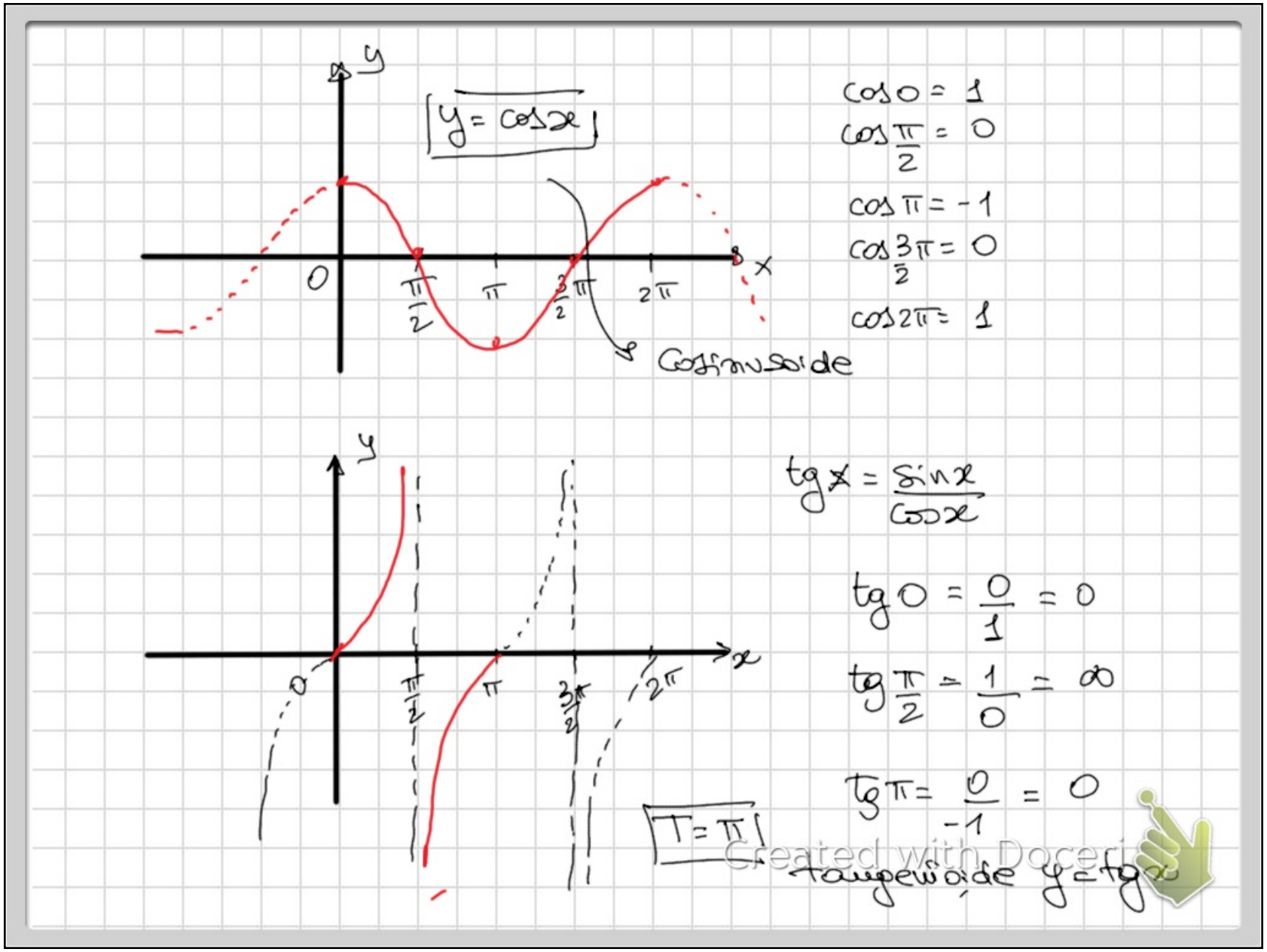
y
y = cos x
0
π/2
π
3/2 π
2π
x
Cosinusoide
cos 0 = 1
cos π/2 = 0
cos π = -1
cos 3/2 π = 0
cos 2π = 1
y
0
π/2
π
3/2
2π
x
T = π
tg x = sin x / cos x
tg 0 = 0/1 = 0
tg π/2 = 1/0 = ∞
tg π = 0/-1 = 0
Tangenoide y = tg x
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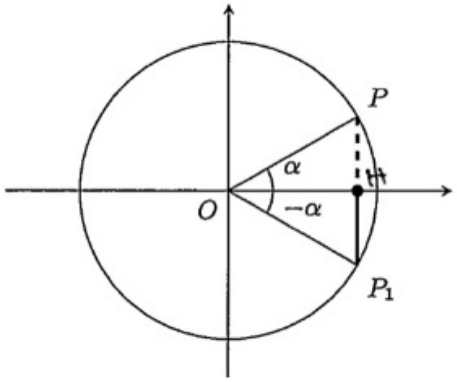
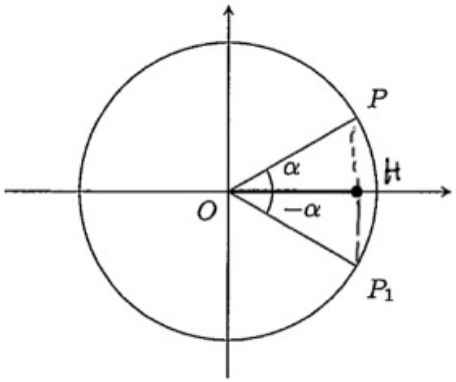
Relazioni goniometriche
Funzioni goniometriche di archi opposti.



$\sin(-\alpha) = -\sin\alpha$



$\cos(-\alpha) = \cos\alpha$

$\sin\alpha = PH$

$\sin(-\alpha) = P_1H$

$\cos\alpha = OH$

$\cos(-\alpha) = OH$

$y = f(x)$

$y = \cos x$ è pari

$f(-x) = -f(x)$

dispari $y = \sin x$ dispari

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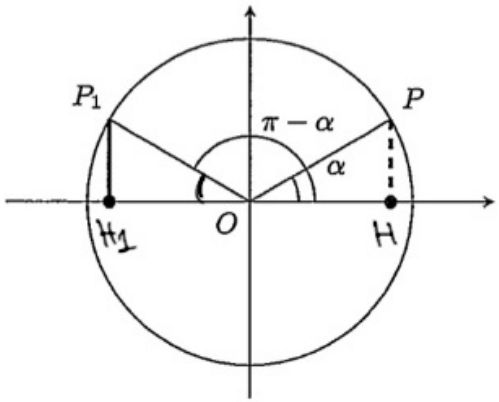
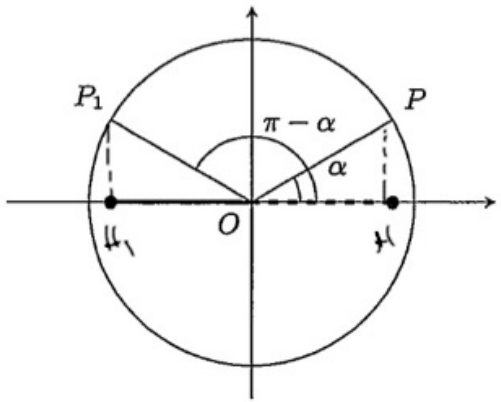
Funzioni goniometriche di archi supplementari



$$\sin(\pi - \alpha) = \sin\alpha$$



$$\cos(\pi - \alpha) = -\cos\alpha$$

$PH = \sin\alpha$

$P_1H_1 = \sin(\pi - \alpha)$ $\quad PH = P_1H_1$

$OH = \cos\alpha$

$OH_1 = \cos(\pi - \alpha)$

$\operatorname{tg}(\pi - \alpha) = -\operatorname{tg}\alpha$

$\operatorname{ctg}(\pi - \alpha) = -\operatorname{ctg}\alpha$

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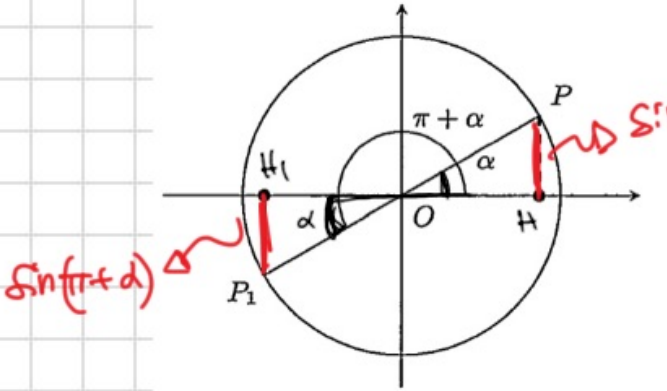
Funzioni goniometriche di archi che differiscono di π



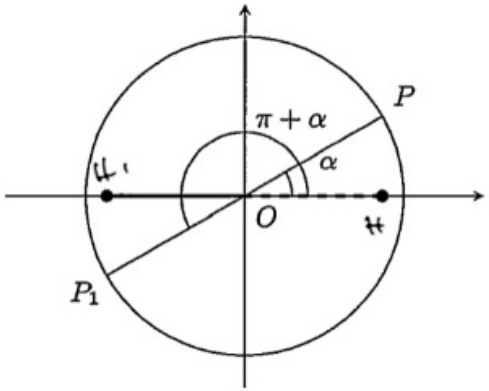
sin α

sin(π+α)

$$\sin(\pi+\alpha) = -\sin\alpha$$

$\overline{PH} = \sin\alpha$

$\overline{P_1H_1} = \sin(\pi+\alpha)$



$$\cos(\pi+\alpha) = -\cos\alpha$$

$\overline{OH} = \cos\alpha$

$\overline{OH_1} = \cos(\pi+\alpha)$

$\mathrm{Tg}(\pi+\alpha) = \mathrm{Tg}\,\alpha$

$\mathrm{Cotg}(\pi+\alpha) = \mathrm{cotg}\,\alpha$

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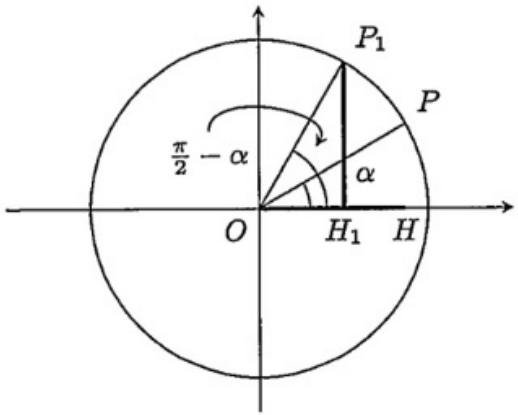
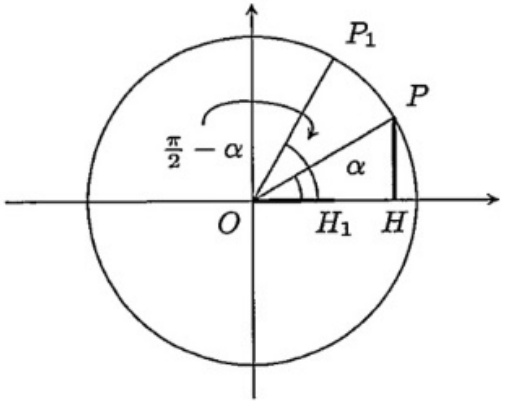
Funzioni goniometriche di archi complementari

(la loro somma è un angolo retto)



$$\sin\left(\frac{\pi}{2} - \alpha\right) = \cos\alpha$$



$$\cos\left(\frac{\pi}{2} - \alpha\right) = \sin\alpha$$

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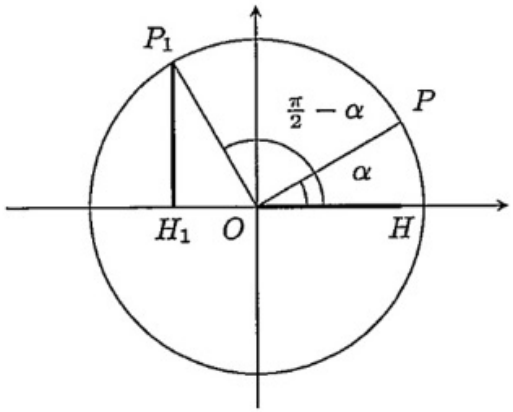
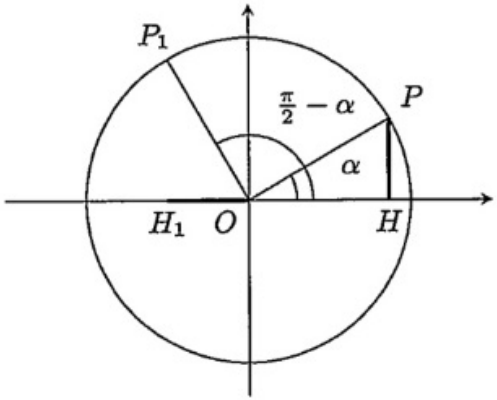
Funzioni goniometriche di archi che differiscono di $\frac{\pi}{2}$



$$\sin\left(\frac{\pi}{2} + \alpha\right) = \cos\alpha$$



$$\cos\left(\frac{\pi}{2} + \alpha\right) = -\sin\alpha$$

$$\text{tg}\left(\frac{\pi}{2} + \alpha\right) = -\text{cotg}\,\alpha$$

$$\text{cotg}\left(\frac{\pi}{2} + \alpha\right) = -\text{tg}\,\alpha$$

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Formule di addizione e sottrazione

$$\sin(\alpha+\beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$$

$$\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$$

$$\operatorname{tg}(\alpha+\beta) = \frac{\operatorname{tg}\alpha + \operatorname{tg}\beta}{1 - \operatorname{tg}\alpha\operatorname{tg}\beta}$$

$$\sin(\alpha-\beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$$

$$\cos(\alpha-\beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$$

$$\operatorname{tg}(\alpha-\beta) = \frac{\operatorname{tg}\alpha - \operatorname{tg}\beta}{1 + \operatorname{tg}\alpha\operatorname{tg}\beta}$$

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Formule di duplicazione

$$\sin 2\alpha = 2 \sin\alpha \cos\alpha$$

$$\cos 2\alpha = \cos^2\alpha - \sin^2\alpha$$

$$\operatorname{tg} 2\alpha = \frac{2 \operatorname{tg}\alpha}{1 - \operatorname{tg}^2\alpha}$$

Formule di bisezione

$$\sin \frac{\alpha}{2} = \pm \sqrt{\frac{1-\cos\alpha}{2}}$$

$$\cos \frac{\alpha}{2} = \pm \sqrt{\frac{1+\cos\alpha}{2}}$$

$$\operatorname{tg} \frac{\alpha}{2} = \pm \sqrt{\frac{1-\cos\alpha}{1+\cos\alpha}}$$

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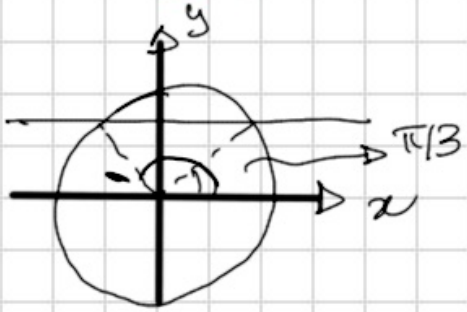
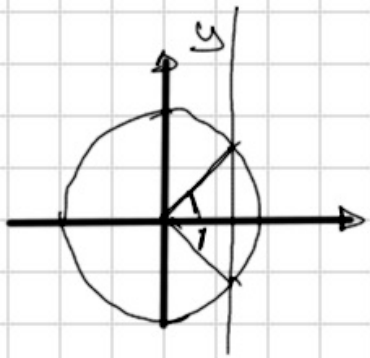
Equazioni goniometriche elementari

$\sin x = \frac{\sqrt{3}}{2}$



$$\begin{cases} x = \frac{\pi}{3} + 2k\pi, \quad k \in \mathbb{Z} \\ x = \pi - \frac{\pi}{3} + 2k\pi, \quad k \in \mathbb{Z} \end{cases} \iff \begin{cases} x = \pi/3 + 2k\pi \\ x = \frac{2}{3}\pi + 2k\pi \end{cases}$$

$\cos x = \frac{\sqrt{2}}{2}$



$$\begin{cases} x = \pi/4 + 2k\pi \\ x = -\frac{\pi}{4} + 2k\pi \end{cases} \quad \text{oppure} \quad \begin{cases} x = \pi/4 + 2k\pi \\ x = 2\pi - \frac{\pi}{4} = \frac{7}{4}\pi \end{cases}$$

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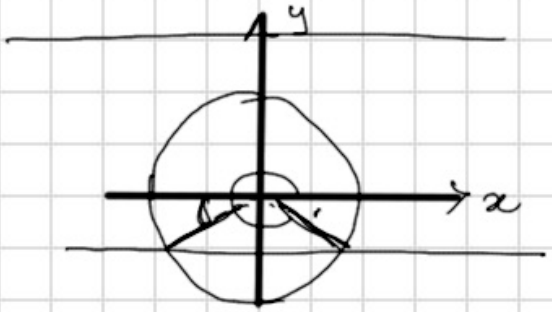
$\sin x = -\frac{1}{2}$

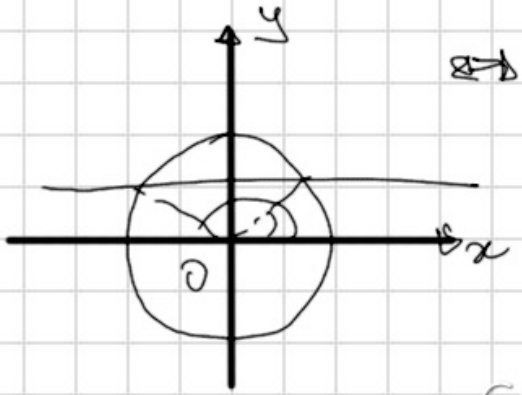


$\sin x = 3$

$-1 \leq \sin x \leq 1$

$$\Leftrightarrow \begin{cases} x = \pi + \frac{\pi}{6} + 2k\pi, & k \in \mathbb{Z} \\ x = 2\pi - \frac{\pi}{6} + 2k\pi, & k \in \mathbb{Z} \end{cases} \Leftrightarrow \begin{cases} x = \frac{7}{6}\pi + 2k\pi \\ x = \frac{11}{6}\pi + 2k\pi \end{cases}$$

$\sin 3x = \frac{\sqrt{2}}{2}$



$$\Leftrightarrow \begin{cases} 3x = \frac{\pi}{4} + 2k\pi \\ 3x = \pi - \frac{\pi}{4} + 2k\pi \end{cases}$$

$$\begin{cases} x = \frac{\pi}{12} + \frac{2}{3}k\pi \\ x = \frac{3}{12}\pi + \frac{2}{3}k\pi \end{cases}$$

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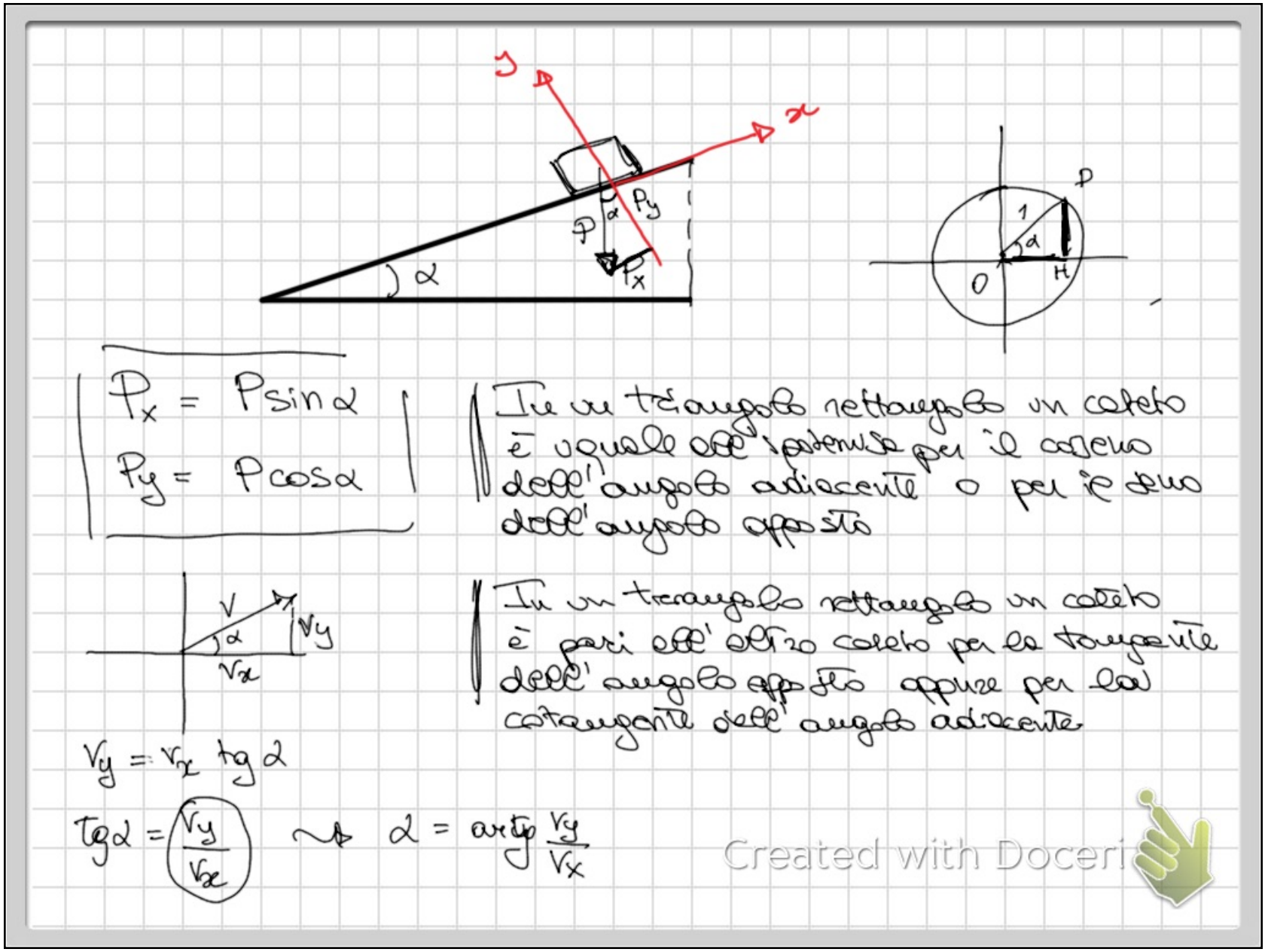
y
x
P_y
P
α
P_x
α
D
1
α
O
H
$P_x = P \sin\alpha$
$P_y = P \cos\alpha$
In un triangolo rettangolo un cateto è uguale all'ipotenusa per il coseno dell'angolo adiacente o per il seno dell'angolo opposto
V
α
V_y
V_x
In un triangolo rettangolo un cateto è pari all'altro cateto per la tangente dell'angolo opposto oppure per la cotangente dell'angolo adiacente
$V_y = V_x \operatorname{tg}\alpha$
$\operatorname{tg}\alpha = \frac{V_y}{V_x} \rightsquigarrow \alpha = \operatorname{arctg}\frac{V_y}{V_x}$
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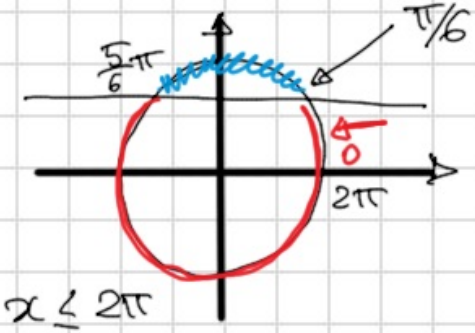
$\sin x \leq \frac{1}{2}$

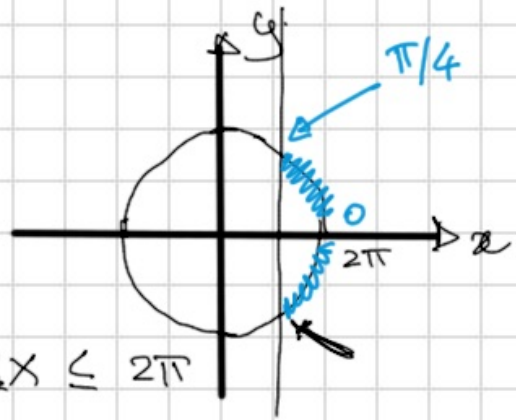


$x \in [0, 2\pi]$

$0 \leq x \leq \frac{\pi}{6} \cup \frac{5\pi}{6} \leq x \leq 2\pi$

$\sin x \geq \frac{1}{2} \iff \frac{\pi}{6} \leq x \leq \frac{5}{6}\pi$

$\cos x > \frac{\sqrt{2}}{2}$



$0 \leq x < \frac{\pi}{4} \cup \frac{7}{4}\pi < x \leq 2\pi$

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