

## TAVOLA DEGLI INTEGRALI INDEFINITI

| Integrazione di funzioni elementari                               | Integrazione di funzioni composte  |
|---|--|
| $\int dx = x + c$   |  |
| $\int x^\alpha dx = \frac{x^{\alpha+1}}{\alpha+1} + c$            | $\int [f(x)]^\alpha \cdot f'(x) dx = \frac{[f(x)]^{\alpha+1}}{\alpha+1} + C$ |
| $\int \frac{1}{x} dx = \ln x  + c$                                | $\int \frac{f'(x)}{f(x)} dx = \ln f(x)  + c$                                 |
| $\int \frac{1}{1+x^2} dx = \arctan x + c$                         | $\int \frac{f'(x)}{1+f^2(x)} dx = \arctan(f(x)) + c$                         |
| $\int \frac{1}{a^2+x^2} dx = \frac{1}{a} \arctan \frac{x}{a} + c$ |  |
| $\int e^x dx = e^x + c$   | $\int e^{f(x)} \cdot f'(x) dx = e^{f(x)} + c$                                |
| $\int a^x dx = \frac{a^x}{\ln a} + c$                             | $\int a^{f(x)} f'(x) dx = \frac{a^{f(x)}}{\ln a} + c$                        |
| $\int \frac{1}{\sqrt{1-x^2}} dx = \arcsin x + c$                  | $\int \frac{f'(x)}{\sqrt{1-[f(x)]^2}} dx = \arcsin f(x) + c$                 |
| $\int -\frac{1}{\sqrt{1-x^2}} dx = \arccos x + c$                 | $\int -\frac{f'(x)}{\sqrt{1-[f(x)]^2}} dx = \arccos f(x) + c$                |
| $\int \cos x dx = \sin x + c$                                     | $\int f'(x) \cdot \cos f(x) dx = \sin f(x) + c$                              |
| $\int \sin x dx = -\cos x + c$                                    | $\int f'(x) \sin f(x) dx = -\cos f(x) + c$                                   |
| $\int \tan x dx = -\ln \cos x  + c$                               |  |

### GLI INTEGRALI INDEFINITI IMMEDIATI

Calcola i seguenti integrali.

|   |   |    |  |
|---|---|----|--|
| 1 | $\int (x^3 + 2x^2 - x - 2) dx$  | 18 | $\int \left( \frac{3}{\sqrt{1-x^2}} - \frac{4}{1+x^2} \right) dx$  |
| 2 | $\int (2x^3 + x^2 - 3x - 3) dx$   | 19 | $\int x^2 \sqrt{x^3 - 1} dx$                                       |
| 3 | $\int \left( \frac{1}{\sqrt{x}} - \frac{1}{x} + \frac{1}{x^2} \right) dx$ | 20 | $\int x^3 \sqrt{x^4 - 2} dx$                                       |
| 4 | $\int \left( \frac{1}{\sqrt{x}} + \frac{1}{x} - \frac{1}{x^3} \right) dx$ | 21 | $\int \frac{7x^6 - 10x^4 - 2x^{-3}}{2x^7 - 4x^5 + 2x^{-2} + 4} dx$ |
| 5 | $\int \frac{x^3 - x^2 - 3x - 2}{x^2} dx$                                  | 22 | $\int \frac{6x^2 - 12x - 6x^{-4}}{x^3 - 3x^2 + x^{-3} + 7} dx$     |

|    |   |    |  |
|----|---|----|--|
| 6  | $\int \frac{2x^3 + 2x^2 - x - 4}{x^2} dx$                         | 23 | $\int e^{\sin x} \cdot \cos x dx$                          |
| 7  | $\int \frac{3x}{\sqrt{1-x^2}} dx$                                 | 24 | $\int -e^{\cos x} \sin x dx$                               |
| 8  | $\int -\frac{x}{2\sqrt{1-x^2}} dx$                                | 25 | $\int \frac{e^x + e^{-x}}{e^x - e^{-x}} dx$                |
| 9  | $\int \frac{x^2}{(x^3 + 1)^2} dx$                                 | 26 | $\int \frac{-2e^{-2x} - e^{-x}}{e^{-2x} + e^{-x}} dx$      |
| 10 | $\int \frac{x^3}{(x^4 - 1)^2} dx$                                 | 27 | $\int \left( \frac{1}{2x-1} + \frac{3}{x+5} \right) dx$    |
| 11 | $\int (2e^x - \cos x + \sqrt[3]{x}) dx$                           | 28 | $\int \left( \frac{4}{3x+2} - \frac{1}{x-1} \right) dx$    |
| 12 | $\int (3e^x - \sin x - \sqrt[3]{x}) dx$                           | 29 | $\int \frac{2x+1}{4x^2-1} dx$                              |
| 13 | $\int \frac{4\sin^3 x + \cos x + \sin(3x)}{\sin x} dx$            | 30 | $\int \frac{x^2 + 3x + 9}{x^3 - 27} dx$                    |
| 14 | $\int \frac{-4\cos^3 x + 2\sin x + \cos(3x)}{\cos x} dx$          | 31 | $\int \frac{x^3}{\sqrt{1-x^8}} dx$                         |
| 15 | $\int \left( \frac{2}{1+x^2} - \frac{5}{\sqrt{1-x^2}} \right) dx$ | 32 | $\int \frac{x^4}{\sqrt{1-x^{10}}} dx$                      |
| 16 | $\int \frac{5^{x+1}}{1+5^{2x}} dx$                                | 33 | $\int \frac{1+e^x}{(x+e^x)\ln(x+e^x)^2} dx$                |
| 17 | $\int \frac{3^{x+2}}{1+3^{2x}} dx$                                | 34 | $\int \frac{2+\cos x}{(1+2x+\sin x)\ln(1+2x+\sin x)^2} dx$ |

### L'INTEGRAZIONE PER PARTI

|   |   |    |                                 |
|---|---|----|---------------------------------|
| 1 | $\int (x+2) \sin x dx$                                    | 9  | $\int \log x dx$                |
| 2 | $\int (x+3) \cos x dx$                                    | 10 | $\int \log^2 x dx$              |
| 3 | $\int x^3 \ln x dx$                                       | 11 | $\int x^2 e^x dx$               |
| 4 | $\int x^4 \ln x dx$                                       | 12 | $\int x \cdot \cos x dx$        |
| 5 | $\int e^{2x} \operatorname{arctg}(e^{-x}) dx$             | 13 | $\int \sin^2 x dx$              |
| 6 | $\int x \operatorname{arctg}\left(\frac{1}{2x}\right) dx$ | 14 | $\int x \cdot \sin x dx$        |
| 7 | $\int 3^x \sin^2(2x) dx$                                  | 15 | $\int \operatorname{tg}^2 x dx$ |
| 8 | $\int 2^x \cos^2(3x) dx$                                  | 16 | $\int x^2 \cos x dx$            |

### L'INTEGRAZIONE DI FUNZIONI RAZIONALI FRATTE

|   |   |    |                                      |
|---|---|----|--------------------------------------|
| 1 | $\int \frac{5x^4 + 2x^2 + 4x}{3x^5 + 2x^3 + 6x^2 - 3} dx$ | 8  | $\int \frac{3x-2}{9x^2 - 6x + 1} dx$ |
| 2 | $\int \frac{x^3 + 2x}{x^4 + 4x^2 + 1} dx$                 | 9  | $\int \frac{2}{x^2 + 8x + 18} dx$    |
| 3 | $\int \frac{3x-1}{x+2} dx$                                | 10 | $\int \frac{1}{4x^2 - 4x + 3} dx$    |
| 4 | $\int \frac{2x-1}{2x+1} dx$                               | 11 | $\int \frac{x+4}{x^2 + 2x + 3} dx$   |
| 5 | $\int \frac{2x-1}{x^2 + 2x - 24} dx$                      | 12 | $\int \frac{x-2}{x^2 + 2x + 5} dx$   |
| 6 | $\int \frac{x-7}{x^2 - 2x - 8} dx$                        | 13 | $\int \frac{2x+1}{x^3 - 1} dx$       |
| 7 | $\int \frac{x+1}{x^2 - 8x + 16} dx$                       | 14 | $\int \frac{x-3}{1-x^3} dx$          |

### L'INTEGRAZIONE PER SOSTITUZIONE

|   |  |
|---|--|
| $\int R(\cos x, \sin x, \operatorname{tg} x) dx,$<br>con $R$ funzione razionale | $\operatorname{tg}\left(\frac{x}{2}\right) = t$<br>$x = 2 \operatorname{arctg} t$<br>$dx = \frac{2}{1+t^2}$<br>$\sin x = \frac{2t}{1+t^2}; \cos x = \frac{1-t^2}{1+t^2}$ |
| $\int R(e^x) dx$  | $e^x = t$<br>$x = \log t$<br>$dx = \frac{1}{t}$  |
| $\int R(\operatorname{tg} x) dx$  | $\operatorname{tg} x = t$<br>$x = \operatorname{arctg} t$<br>$dx = \frac{1}{1+t^2}$  |
| $\int R(\sqrt{x}) dx$   | $\sqrt{x} = t$<br>$x = t^2$<br>$dx = 2t$   |

|   |  |    |   |
|---|--|----|---|
| 1 | $\int \frac{1}{1 - \cos x} dx$                   | 9  | $\int \frac{2}{1 + \sin x - 2 \cos x} dx$ |
| 2 | $\int \frac{\sin x}{1 + \sin x} dx$              | 10 | $\int \frac{2}{3 + \sqrt{x}} dx$          |
| 3 | $\int \frac{\operatorname{tg} x}{1 - \cos x} dx$ | 11 | $\int \frac{2}{e^x + e^{-x}} dx$          |

|   |  |    |   |
|---|--|----|---|
| 4 | $\int \frac{dx}{\cos x \cdot \sin^2 x}$          | 12 | $\int \frac{x \cos \sqrt{x^2 + 1}}{\sqrt{x^2 + 1}} dx$      |
| 5 | $\int \frac{\cos x}{\cos^2 x - 2 \sin x - 3} dx$ | 13 | $\int \sqrt[3]{e^{2x}} (e^x + 1) dx$                        |
| 6 | $\int \frac{dx}{1 + \sin x + \cos x}$            | 14 | $\int \frac{3e^{2x} + 2e^x}{2e^{2x} + 3e^x + 1} dx$         |
| 7 | $\int \frac{1 + \sin x}{1 - \sin x} dx$          | 15 | $\int \frac{e^{2x}}{3e^{2x} - 2e^x - 1} dx$                 |
| 8 | $\int \frac{3 + \operatorname{tg} x}{\cos x} dx$ | 16 | $\int \frac{\sqrt{x} + 3}{\sqrt{x}(2x - 3\sqrt{x} - 2)} dx$ |

### IL TEOREMA FONDAMENTALE DEL CALCOLO INTEGRALE

|    |  |  |
|----|--|--|
| 1  | $\int_1^2 (x^3 - x + 2) dx$  | $\left[ \frac{17}{4} \right]$                      |
| 2  | $\int_1^2 (x^3 + x - 1) dx$  | $\left[ \frac{17}{4} \right]$                      |
| 3  | $\int_1^4 \frac{2x^2 + 3}{x} dx$   | $[15 + 3 \ln 4]$                                   |
| 4  | $\int_1^3 \frac{3x^3 + 2}{x} dx$   | $[26 + 2 \ln 3]$                                   |
| 5  | $\int_0^{\frac{\pi}{4}} (\sin x - \cos x) dx$                            | $[1 - \sqrt{2}]$                                   |
| 6  | $\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} (\cos x - \sin x) dx$              | $[1 - \sqrt{2}]$                                   |
| 7  | $\int_0^2 x e^{x^2} dx$  | $\left[ \frac{e^4 - 1}{2} \right]$                 |
| 8  | $\int_0^1 x^2 e^{x^3} dx$  | $\left[ \frac{e - 1}{3} \right]$                   |
| 9  | $\int_{-1}^1 \frac{2x^3 - 3x^2 - 10x + 9}{x + 2} dx$                     | $\left[ \frac{28}{3} + \ln 3 \right]$              |
| 10 | $\int_3^6 \frac{x^3 + 2x^2 - 2x + 2}{x - 1} dx$                          | $\left[ \frac{213}{2} + 3 \ln \frac{5}{2} \right]$ |
| 11 | $\int_0^{\frac{1}{2}} \frac{2x}{\sqrt{1-x}} dx$                          | $\left[ \frac{8 - 5\sqrt{2}}{3} \right]$           |
| 12 | $\int_0^{\frac{1}{2}} \frac{2x}{\sqrt{1+x}} dx$                          | $\left[ \frac{8 - 3\sqrt{6}}{3} \right]$           |
| 13 | $\int_{\frac{\pi}{2}}^{\frac{3\pi}{4}} \frac{\sin x}{1 + 2 \cos^2 x} dx$ | $\left[ \frac{\pi \sqrt{2}}{8} \right]$            |

|    |  |   |
|----|--|---|
| 14 | $\int_0^{\frac{\pi}{4}} \frac{\cos x}{1+2\sin^2 x} dx$ | $\left[ \frac{\pi\sqrt{2}}{8} \right]$                |
| 15 | $\int_0^{\frac{\pi}{4}} (x+1)\sin x dx$                | $\left[ 1 - \frac{\pi\sqrt{2}}{8} \right]$            |
| 16 | $\int_0^{\frac{\pi}{4}} (x+1)\cos x dx$                | $\left[ \frac{\pi\sqrt{2}}{8} + \sqrt{2} - 1 \right]$ |

### IL CALCOLO DELLE AREE DI SUPERFICI PIANE

Disegna le superfici delimitate dall'asse  $x$  e dal grafico delle funzioni seguenti, definite negli intervalli indicati, poi calcolane l'area.

|   |  |  |
|---|--|--|
| 1 | $y = e^x + 1, [0; 2].$   | $\left[ e^2 + 1 \right]$               |
| 2 | $y = e^x + 2, [0; 1].$   | $\left[ e + 1 \right]$                 |
| 3 | $y = -x^2 + 2x, [-1; 2].$  | $\left[ \frac{8}{3} \right]$           |
| 4 | $y = -x^2 - 2x, [-2; 1].$  | $\left[ \frac{8}{3} \right]$           |
| 5 | $y = x^3 - 6x^2 + 11x - 6, [1; 3]$   | $\left[ \frac{1}{2} \right]$           |
| 6 | $y = x^3 - 3x^2 + 2x, [0; 2].$   | $\left[ \frac{1}{2} \right]$           |
| 7 | Determina l'area della regione finita di piano delimitata dalla retta di equazione $2x + 2y - 9 = 0$ e dall'iperbole di equazione $y = \frac{2}{x}$ .  | $\left[ \frac{63}{8} - 3\ln 4 \right]$ |
| 8 | Determina l'area della regione finita di piano delimitata dalla retta di equazione $2x - 2y - 9 = 0$ e dall'iperbole di equazione $y = -\frac{2}{x}$ . | $\left[ \frac{63}{8} - 3\ln 4 \right]$ |